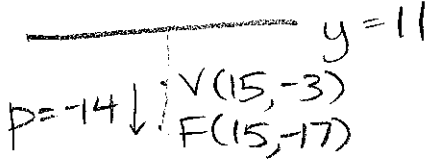
Find the equation of the parabola with focus $(15, -17)$ and directrix $y = 11$.

SCORE: ____ / 15 PTS



$\text{VERTEX} = \left(15, \frac{-17+11}{2}\right) = (15, -3)$

$p = -17 - -3 = -14$

$(x-15)^2 = 4(-14)(y+3)$

$(x-15)^2 = -56(y+3)$

Consider the conic with the polar equation $r = \dfrac{36}{7 - 5\cos\theta}$.

SCORE: _____ / 30 PTS

[a] What is the shape of the graph of the equation ?

$r = \dfrac{\frac{36}{7}}{1 - \frac{5}{7}\cos\theta}$ $\quad e = \frac{5}{7} < 1 \longrightarrow$ ELLIPSE

[b] Find the equation of the directrix.

$ep = \frac{36}{7}$

$\frac{5}{7}p = \frac{36}{7}$ $\quad x = -\frac{36}{5}$

$p = \frac{36}{5}$

[c] Find the rectangular co-ordinates of the focus/foci, using the process in the lecture & website handout.

θ	r	(x, y)	
0	18	$(18, 0)$	← VERTICES
$\frac{\pi}{2}$	$\frac{36}{7}$	$(0, \frac{36}{7})$	
π	3	$(-3, 0)$	← VERTICES
$\frac{3\pi}{2}$	$\frac{36}{7}$	$(0, -\frac{36}{7})$	

CENTER $= \left(\frac{18-3}{2}, 0\right)$

$= \left(\frac{15}{2}, 0\right)$

FOCI $= (15, 0), (0, 0)$

Consider the polar equation $r = -2 - 3\sin\theta$.

SCORE: ____ / 30 PTS

The following symmetry tests all fail: $(-r, \theta)$, $(-r, -\theta)$ and $(-r, \pi - \theta)$

[a] Run the other standard tests for symmetry for the polar equation, and summarize all conclusions in the table below.

POLAR AXIS: $(r, -\theta)$

$r = -2 - 3\sin(-\theta)$

$r = -2 + 3\sin\theta$

SYMMETRIC OVER POLAR AXIS ?	NO CONCLUSION
SYMMETRIC OVER POLE ?	NO CONCLUSION
SYMMETRIC OVER $\theta = \frac{\pi}{2}$?	YES

POLE: $(r, \pi + \theta)$

$r = -2 - 3\sin(\pi + \theta)$

$r = -2 - 3[\sin\pi\cos\theta + \cos\pi\sin\theta]$

$r = -2 + 3\sin\theta$

$\theta = \frac{\pi}{2}$: $(r, \pi - \theta)$

$r = -2 - 3\sin(\pi - \theta)$

$r = -2 - 3[\sin\pi\cos\theta - \cos\pi\sin\theta]$

$r = -2 - 3\sin\theta$

[b] What is the minimum interval of θ – values that must be plotted before using symmetry to complete the graph ?

$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

[c] Name the shape of the graph of the polar equation. If the graph is a rose curve, state the number of petals.

$|-2| < |-3| \rightarrow$ LIMAÇON WITH LOOP

Consider the ellipse with foci $(-7, -2)$ and $(-7, 4)$ and minor axis of length 12. **SCORE: ____ / 20 PTS**

[a] Find the equation of the ellipse.

$\text{CENTER} = \left(-7, \frac{-2+4}{2}\right) = (-7, 1)$

$2b = 12 \longrightarrow b = 6$

$c = 4 - 1 = 3$

$a^2 = 6^2 + 3^2 = 45$

- $F(-7,4)$
- $C(-7,1)$
- $F(-7,-2)$

$$\frac{(x+7)^2}{6^2} + \frac{(y-1)^2}{45} = 1 \longrightarrow \frac{(x+7)^2}{36} + \frac{(y-1)^2}{45} = 1$$

[b] Find the co-ordinates of the vertices.

$(-7, 1 \pm 3\sqrt{5})$

Convert the rectangular equation $y = 2x - 3$ to polar.

SCORE: ____ / 15 PTS

$$r\sin\theta = 2r\cos\theta - 3$$

$$r\sin\theta - 2r\cos\theta = -3$$

$$r(\sin\theta - 2\cos\theta) = -3$$

$$r = -\frac{3}{\sin\theta - 2\cos\theta}$$

$$r = \frac{3}{2\cos\theta - \sin\theta}$$

Consider the conic with equation $3x^2 - 2y^2 + 6x + 12y - 3 = 0$. **SCORE: _____ / 30 PTS**

[a] Find the co-ordinates of the focus/foci.

$3x^2+6x-2y^2+12y=3$

$3(x^2+2x+1)-2(y^2-6y+9)=3+3-18$

$3(x+1)^2-2(y-3)^2=-12$

$\therefore \quad \frac{(y-3)^2}{6}-\frac{(x+1)^2}{4}=1 \qquad c^2=6+4=10$

$(-1, 3\pm\sqrt{10})$

[b] If the equation corresponds to a circle, find its radius.
If the equation corresponds to a parabola, find its vertex & directrix.
If the equation corresponds to an ellipse, find the endpoints of its minor axes.
If the equation corresponds to a hyperbola, find the equations of the asymptotes.

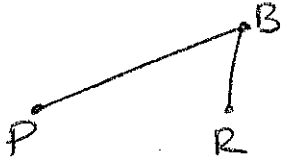
$y-3=\pm\frac{\sqrt{6}}{2}(x+1)$

Pat & Reese live in separate houses on Taylor Street.
Taylor Street is parallel to Jordan Road. Both roadways are straight.

SCORE: _____ / 10 PTS

Draw diagrams and write algebraic equations involving distances to answer the following questions.

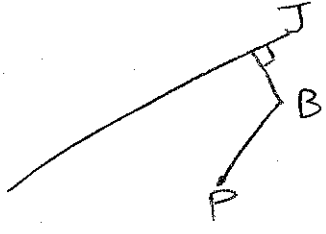
[a] A bird is flying through the neighborhood. The distance from Pat's house to the bird to ~~Chris~~ REESE's house is always 300 meters. What is the shape of the bird's path ? (Assume the bird is flying at a constant low height.)



$BP + BR = 300$

ELLIPSE

[b] There is a bike path nearby. Every point on the path is half as far from Jordan Road as it is from Pat's house. What is the shape of the bike path ?



$BJ = \frac{1}{2} BP$

$e = 2 = \frac{BP}{BJ}$ HYPERBOLA